AN APPROXIMATE SOLUTION FOR PERPETUAL AMERICAN OPTION WITH TIME TO BUILD: THE VALUE OF ENVIRONMENTAL REMEDIATION INVESTMENT PROJECTS

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ABSTRACT

Most investors and real estate developers consider brownfield redevelopment projects risky and, to compensate for the additional environmental risk, demand higher returns on the investment needed to cleanup and redevelop a contaminated property. The perception of risk is aggravated by the fact that, depending upon the remediation technique adopted, remediation of a contaminated site takes time during which, the real estate market conditions may change significantly.

To estimate the value of brownfields considering the environmental and market risks associated with property clean up and redevelopment, both technical and market risks are integrated. A closed-form solution derived for a perpetual American call option was modified to calculate the optimal sequential investment that accounts for the required time to undertake the investment (i.e., implement the proposed remediation for a contaminated land). The proposed solution can be used to evaluate the value of a brownfield if the owner/developer has the option to delay remediation indefinitely waiting for optimal conditions to start. The results of the proposed approximation are compared to the results obtained using numerical techniques to solve partial differential equations.

INTRODUCTION

Quantification of risk and economic cost associated with actual and/or potential environmental pollution is central to the success of brownfield redevelopment. The term brownfield refers to abandoned, idled, or underutilized environmentally impaired properties. A brownfield redevelopment includes cleaning the site up so it can be put back to a productive use for residential, commercial, and/or industrial purposes. As it is the case with any construction project, remediation of a contaminated site takes time to complete. Different from typical construction projects, real estate transactions that includes cleaning of a contaminated land is usually considered risky by most investors and demand a higher return on investment in order to take the project and the rate of return demanded for each project depends upon each investors risk preference.

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Although remediation of contaminated sites in the United States are enforced by local or federal agencies, many sites are cleaned voluntarily without a specific time table. This may be the case for which the owner/developers, who are not responsible for the site contamination and they have bought the property with the intent to redevelop it as soon as the real estate conditions are appropriate. For these situations, voluntary clean up (remediation) of contaminated properties can be viewed as a perpetual American call option on the clean property; that is the owner or developer has the right, but not the obligation, to pay a total sunk remediation cost in return for a real estate project (clean property).

PRELIMANARY FORMULATION

Because remediation of a contaminated site takes time, as it is the case with any investment project, a straight forward application of the solution to the perpetual American call option (Samuelson, 1965) cannot be applied. The solution to this problem was provided by Majd and Pindyck (1987). They analyzed the problem of investing continuously and optimally until the project is completed assuming that investment can be stopped and restarted at no cost.

The value of the cleaned land (V) is assumed to follow a geometric Brownian motion process of the form:

$$dV = \alpha V dt + \sigma V dz \tag{1}$$

where α is the expected rate of growth; σ is the uncertainty associated to the value of the clean property; dz is the differential of a standard Wiener process (with mean 0 and variance dt). The exercise price (i.e., the total remaining remediation cost, K) is spend over a period of time at a maximum rate of k. The total remaining remediation cost (K) and the rate of expenditure (k) are related as follows:

$$dK = \begin{cases} -kdt & \text{for} \quad V \geq V^*(K) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $V^*(K)$ represents the optimal investment value of the cleaned property as a function of the remaining remediation cost. Equation (2) indicates that investment only takes place if the value of the cleaned property is greater than the optimal value, $V^*(K)$, which needs to be obtained as part of the solution. The investment problem above described can be written in a partial differential form as:

for $V < V^{*}(K)$ $$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 C}{\partial V^2} + (r-\delta)V\frac{\partial C}{\partial V} - rC = 0 \tag{3a}$$

for $V \geq V^{*}(K)$ $$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 C}{\partial V^2} + (r-\delta)V\frac{\partial C}{\partial V} - rC - k\frac{\partial C}{\partial K} - k = 0 \tag{3b}$$

where C is option value of the cleaned property; r is the risk-free discount rate; $\delta = \mu - \alpha$ ($\mu > \alpha$) is the opportunity cost; μ is the market risk-adjusted expected rate of return from owning the property; K is the total remaining expenditure; k is the rate of expenditure. Equation (3a) can be solved analytically whereas Equation (3b) must be solved numerically using finite different techniques (e.g., Majd and Pindyck, 1987).

APPROXIMATE SOLUTION

As discussed above, voluntary clean up (remediation) of contaminated properties can be viewed as a perpetual American call option on the clean property; that is the owner or developer has the right, but not the obligation, to pay a total sunk remediation cost (K) spent a certain rate in return for a real estate project (clean property) whose current value is V (V is stochastic and changes with time). If the remediation cost is assumed to be spent instantaneously, the closed-form solution for the perpetual American call option on a dividend-paying stock derived by Samuelson (1965) could be used to estimate the value of the investment:

$$\beta = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{4}$$

$$V* = \frac{\beta}{\beta - 1} K_o \tag{5}$$

$$A = \frac{V* - K_o}{(V*)^{\beta}} \tag{6}$$

$$C(V) = \begin{cases} V_o - K_o & \text{for} \quad V \geq V^{*}(K) \\ A V_o^{\beta} & \text{otherwise} \end{cases} \tag{7}$$

In order to use the closed form solution derived for the American perpetual call option given by Equations (4) to (7), equivalents investment cost and value of the real estate investment need to be used. The expressions that were found to provide consistently equivalent results of the optimal value of the real estate investment and the option are: (i) the present value of the total remediation cost; and (ii) the present value of the real estate investment. The present value of the investment cost (K_o) is obtained assuming that investment is made continuously over the time period T=K/k such that:

$$K_o = \int_0^T ke^{-rt} dt = (1 - e^{-rK/k}) \frac{k}{r} \tag{8}$$

Similarly, The present value of the real estate investment (V_o) is obtained assuming that investment cost is made continuously over the time period T=K/k until the remediation is completed. The present value of the completed project (i.e., cleaned land) V that is expected to grow at a rate α can be estimated as using the discount rate μ as:

$$V_o = \left(Ve^{\alpha K/k} \right) e^{-\mu K/k} = Ve^{-\delta K/k} \tag{9}$$

Equations (8) and (9) can be used together with Equations (4) through (7) to obtain the value of an investment that takes time to build.

VALIDATION

Equations (4) through (9) can be used to evaluate the option value of an asset that takes time to build (e.g., the remediation of a contaminated site) . In this section, the proposed simplified equations are validated against the results of the more rigorous numerical solution provided by Majd and Pindyck (1987). Table 1 shows the results of the value of the option to invest (C) assuming an annual risk-free rate r = 2%, annual rate of opportunity cost δ = 6%, and annual standard deviation σ = 20%, a rate of expenditure k = 1 million/year, for K = 1, 2, 3, 4, 5, and 6 million and selected values of V (i.e., values are selected from Majd and Pindyck, 1987). Table 1 also shows the results from Majd and Pindyck (in italics) obtained using numerical finite difference techniques. As shown in the table, the results of the proposed approximate solution agrees well with the numerical results. The average error between the two sets of results shown in Table 1 is 0.79 percent.

According to the results provided by the finite difference solution and summarized in Table 1, for a total remaining remediation cost of K=\$3 million, investment should continue if the value of the real estate (V) is greater than V^* = \$5.21 million (i.e., the optimal option value C^* = \$1.42 million). A value of C(V=5.21, K=3)=\$1.44 million using the approximate solution is shown in Table 1 (i.e., a 1.5% difference with the numerical solution). This value, however, does not correspond top the optimal call option value as discussed below.

The values of the optimal call option value, C^* can be calculated using Equations (4) through (9). For instance, for a total remaining remediation cost of K=$3 million, using Equations (5) and (7), the value of the optimal real estate investment and its corresponding option value are V^* = $5.0 million and C^* = $1.26 million, respectively. For this case, the difference between the option values calculated using the approximate solution and the numerical scheme is $0.16 million (i.e., that is 12.7%) whereas the difference between the optimal real estate investment is $0.21 million (i.e., that is 4.8%). A comparison of the optimal call option values (C^*) for each value of total remaining cost (K) obtained using the proposed approximated solution and the results from the numerical solution by Majd and Pindyck (1987) is presented in Figure 1.

Table 1 – Option Investment Value, $C^{1,2}$

Value of Completed Project, V (millions)	Total Remaining Investment (K), millions					
	6	5	4	3	2	1
20.09	8.36 *8.22*	10.12 *10.00*	11.96 *11.85*	13.87 *13.78*	15.86 *15.79*	17.93 *17.89*
14.88	4.73 *4.63*	6.27 *6.18*	7.86 *7.78*	9.52 *9.46*	11.24 *11.19*	13.02 *13.00*
11.02	2.06 ***2.02***	3.41 *3.34*	4.82 *4.77*	6.29 *6.25*	7.81 *7.79*	9.39 *9.37*
9.49	1.26 *1.22*	2.27 ***2.23***	3.62 *3.57*	5.01 *4.98*	6.46 *6.43*	7.95 *7.93*
7.03	0.47 *0.44*	0.85 *0.81*	1.69 ***1.65***	2.96 *2.93*	4.27 *4.26*	5.63 *5.62*
5.21	0.17 *0.18*	0.31 *0.29*	0.63 *0.60*	1.44 ***1.42***	2.66 *2.65*	3.92 *3.91*
3.32	0.04 *0.04*	0.07 *0.06*	0.14 *0.13*	0.33 *0.31*	0.98 ***0.98***	2.14 *2.13*
2.86	0.02 *0.02*	0.04 *0.04*	0.09 *0.08*	0.20 *0.19*	0.61 *0.59*	1.70 *1.70*
1.82	0.01 0.00	0.01 0.01	0.02 0.02	0.04 0.04	0.14 0.13	0.72 **0.73**

1 Values in italics were obtained from Majd and Pindyck (1987).

2 Values in bold and italics represent the optimal investment value, C^*, that correspond to each value of K. That is, investment takes place only if V is greater than the corresponding optimal value.

Although the calculated option value for each pair (V,K) in Table 1 agrees well with the ones calculated using the numerical finite difference scheme, as shown in Figure 1, the optimal option value may not be accurately calculated using the numerical algorithm scheme.

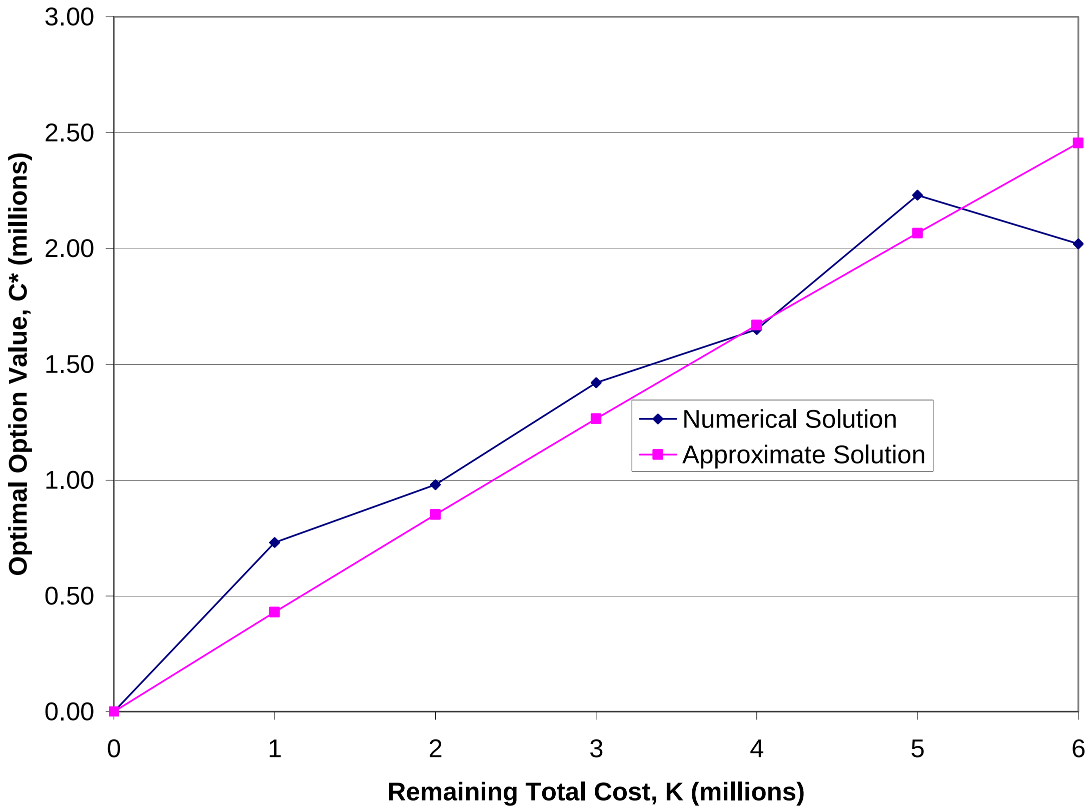

Figure 1 – Comparison of Optimal Option Value (C^*)

The main reason for the discrepancy is the fact that the calculated values of the option $C(V,K)$ using the numerical algorithm are obtained at the discrete values of V shown in Table 1 may not necessarily include the optimal values V^*. Each of the values of V selected for the numerical analysis were obtained as follows (Majd and Pindyck, 1987):

$$V = e^{0.15j} \tag{10}$$

for $j = 0, 1, 2, \ldots 25$. If follows from Equation (10) that as j grows, the increment in V grows exponentially. A better agreement would likely be obtained if a finer mesh is used to for the finite different scheme.

On the other hand, as shown in Figure 2, the agreement between the calculated optimal value of completed *real* estate project using the approximate solution and the numerical finite different scheme is very good. Therefore, the optimal value for starting the remediation project can be reasonable estimated using the approximate solution presented in this paper.

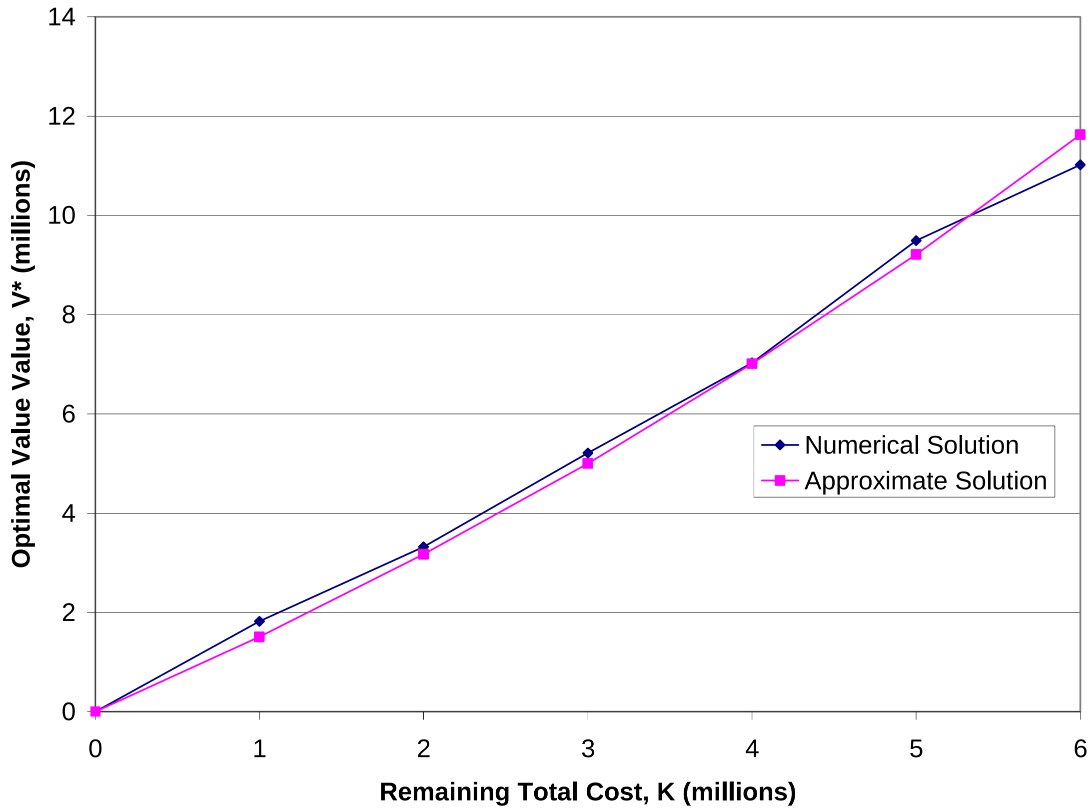

Figure 2– Comparison of Optimal Value of Completed Project (V*)

APPLICATION

For the environmental application discussed herein, the proposed modified American perpetual option presented in this paper can be used to answer the following questions:

1. What is the value of a real estate investment (call option value, C*) if the land is contaminated and prospective seller or buyer can remediate it at any time?
2. What is the optimal value of the project (V^*) at which a developer should start remediation of the site and pay the remediation cost K over a period of time?
3. How these values are affected if the remediation cost is uncertain?

Assuming that the average cost to remediate the contaminated land is $5 million, the current value of the cleaned land is 6 million, the market volatility of the real estate project is 20 percent, the interest rate is two percent, the average time to redevelop is three years, and the project revenue stream, once it is completed, is six percent of the current value, using Equations (4) to (9), the estimated value of the optimal investment (V*) and the option to wait are $6.96 and $0.71, respectively. Because the current value of the land is 6 million, it is optimal to wait.

Remediation project are uncertain regarding cost and time to implement the remediation. In general, a probability distribution can be estimated based upon technical information of the site and the remediation technique. These uncertainties can be integrated to the uncertainty of the market value of the cleaned property using the model presented in this paper along with Monte Carlo simulations. To this end, the commercial software Crystal Ball^{TM} is used to conduct the simulations. For this case, the remediation cost and the time to complete the remediation project are both considered random correlated variables log normally distributed with a mean (and standard deviation) values of $5 ($2.35) million and 3 (1.2) years, respectively, and a coefficient of correlation of 0.8 (Figures 3 and 4). The uncertainty associated with the real estate market is already accounted for in Equations (4) through (9).

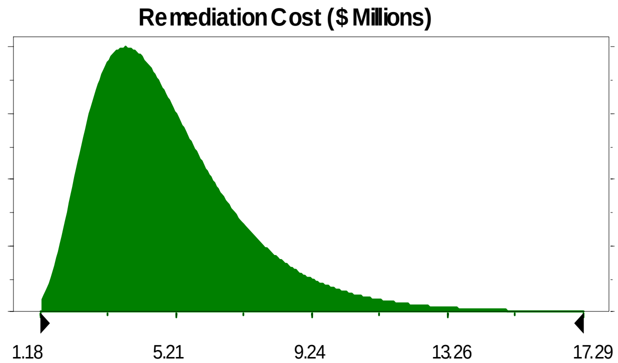

Figure 3 – Probability Distribution for Remediation Cost

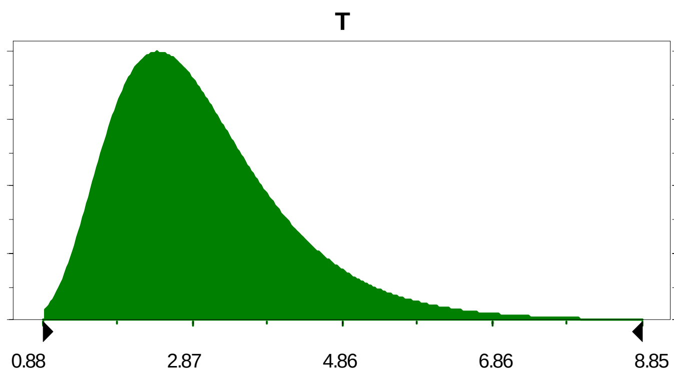

Figure 4 – Probability Distribution for Time to Completion

The calculated optimal value to start the remediation project has the distribution shown in Figure 5 having a mean of $8.6 million and a standard deviation of $4.4 million. The optimal net present value of the contaminated property (i.e., the perpetual American call option) is $3.1 million (Figure 6) having a standard deviation of $1.7 million (assuming that the developer can wait for this optimal value). For this case, because the uncertainty in the remediation cost and the time to complete is high, the optimal value to start remediating the site is about 23 percent higher than the case without uncertainty of these parameters. Thus, as expected, technical uncertainty favors delaying the project.

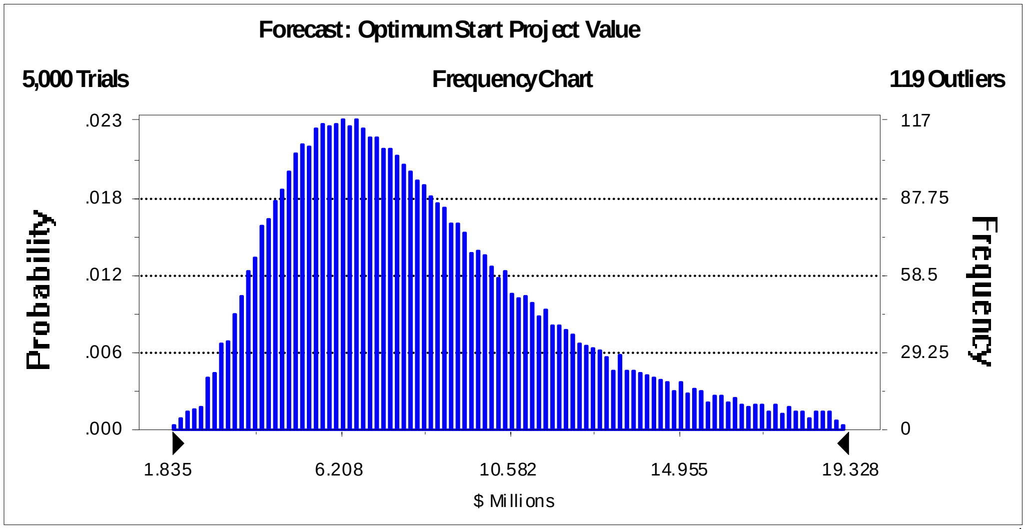

Figure 5 – Probability Distribution for Optimum Project Value (V^*)

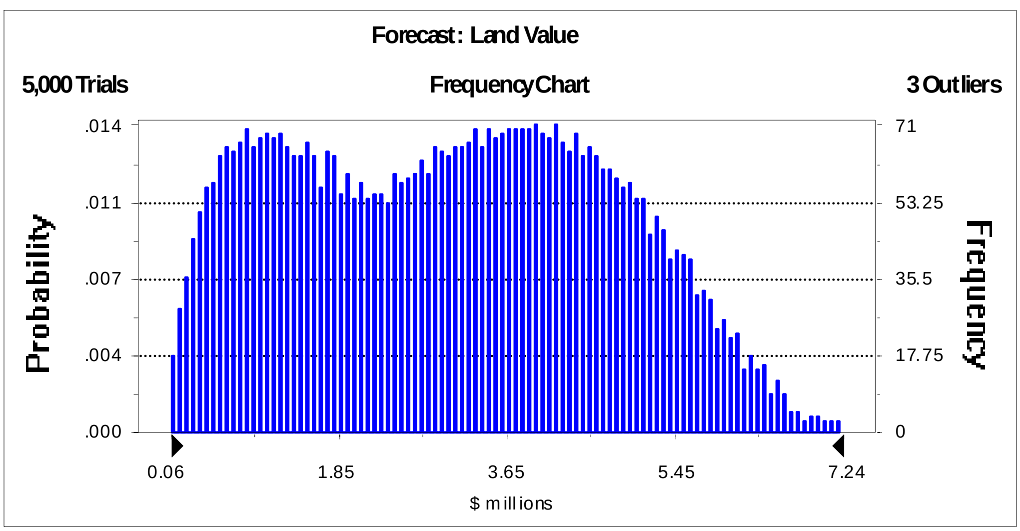

Figure 4 – Probability Distribution for Option Value

CONCLUSIONS

A simplified model to calculate the optimal option value for projects that require time to implement has been presented in this paper. The results of the simplified model has been successfully compared to the results obtained using numerical techniques. For the environmental application discussed herein, the proposed modified American perpetual option presented in this paper can be used to answer questions regarding the value of brownfield projects taking into consideration technical and market uncertainty.

ACKNOWLEDGEMENTS

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